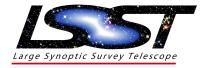




Photo-*z* PDF tests and storage

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LSST Project will provide. . .

as many point estimates and PDFs as can fit in 200 floats! (See <u>https://ldm-151.lsst.io/</u> for more details.)

With input from LSST-DESC, LSST Project must choose... how many and which point estimators how many and which PDFs in what parametrizations with how many parameters

Goal: Develop a metric to optimize these choices!



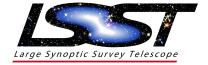
Seek to encapsulate each p(z) in N floats \vec{d} for storage then reconstruct $\hat{p}(z)$ from \vec{d} for science use later

Formats considered

- Samples $\{\hat{z}_s\}_N$ for $z_s \sim p(z)$
- Binned (histogram) $\{\hat{P}(z_b)\}_N$ for $\hat{P}(z_b) = CDF_p[z_b] CDF_p[z_{b-1}]$
- Quantiles $\{\hat{z}_n\}_N$ for $\hat{z}_n = CDF_p^{-1}[q_n]$
- Gridded $\{p(z_g)\}_N$
- Functional Parameters $\{\theta_i\}_N$ for $f_{\{\theta_i\}_N}(z) = p(z)$

 $\circ f$ may be any specified mixture model, polynomial, spline, etc.

$$\operatorname{recall} CDF_p[z] = \int^z p(z')dz'$$



Root-mean-square error (RMSE)

$$D_{RMSE} = \sqrt{\frac{1}{T} \sum_{t}^{T} (\hat{p}(z_t) - p(z_t))^2}$$

Kullback-Leibler (KL) divergence

$$D_{KL} = \sum_{t}^{T} p(z_t) \log \left[\frac{p(z_t)}{\hat{p}(z_t)} \right]$$

Note asymmetry!

See https://github.com/aimalz/qp/blob/master/docs/notebooks/kld.ipynb

Kolmogorov-Smirnov (KS) test statistic

$$D_{KS} = \max_{t} |CDF_{\hat{p}}[z_t] - CDF_{p}[z_t]|$$

Anderson-Darling (AD) test statistic

$$S_{AD} = \sum_{t}^{T} \int_{\infty}^{\infty} \left(CDF_{\hat{p}}[z_t] - CDF_{p}[z_t] \right)^2 w(z_t) dCDF_{p}[z_t]$$

Cramer-von Mises (CvM) test statistic $w_{CvM}(z_t) = 1$

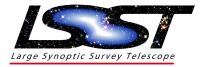


Quantile-quantile (Q-Q) plot

 $\{(CDF_p(z_t), CDF_{\hat{p}}(z_t))\}_T$

Comparison of moments $\{(\mu_m, \hat{\mu}_m)\}_M$

Recall
$$\mu_m = \sum_t^T z_t^m p(z_t)$$
 and $\hat{\mu}_m = \sum_t^T z_t^m \hat{p}(z_t)$



(Malz & Marshall, in prep.)

qp is...

a framework for optimizing the choices of1D PDF parametrization and number of parametersfor storage and distribution

qp is <u>not</u>...

an "answer" to any question, particularly how many and which PDF methods to provide in data releases



(Malz & Marshall, in prep.)

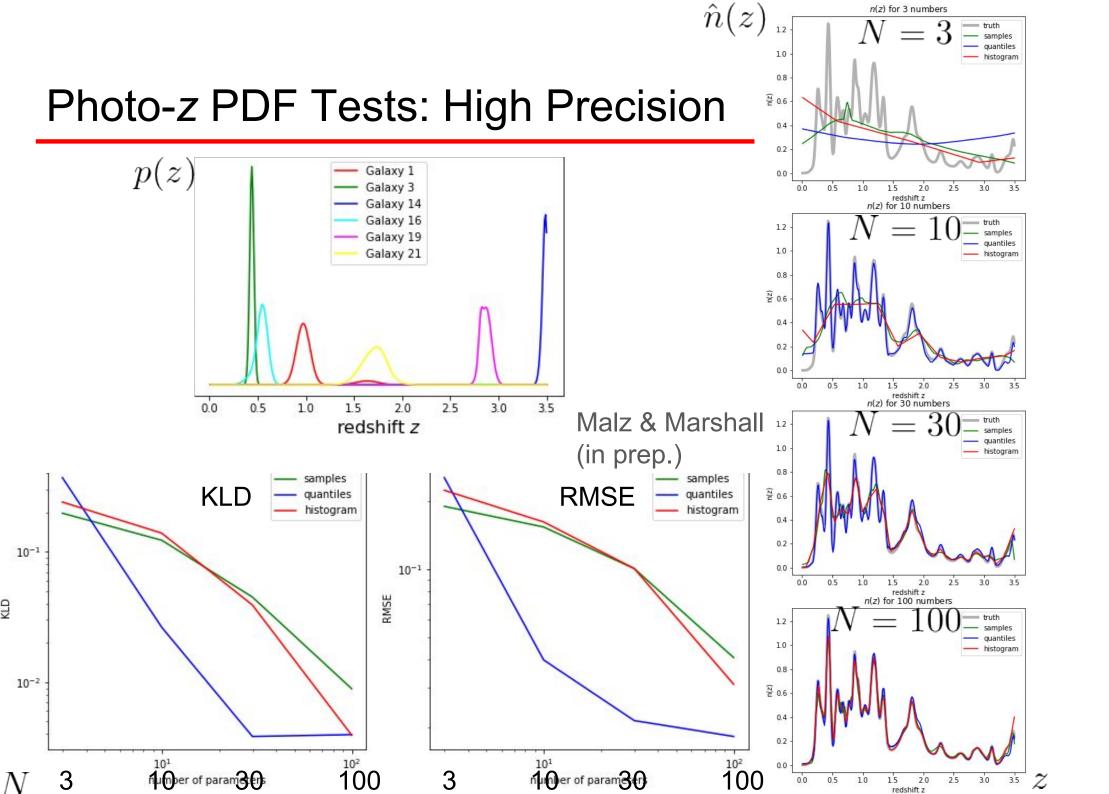
Test procedure

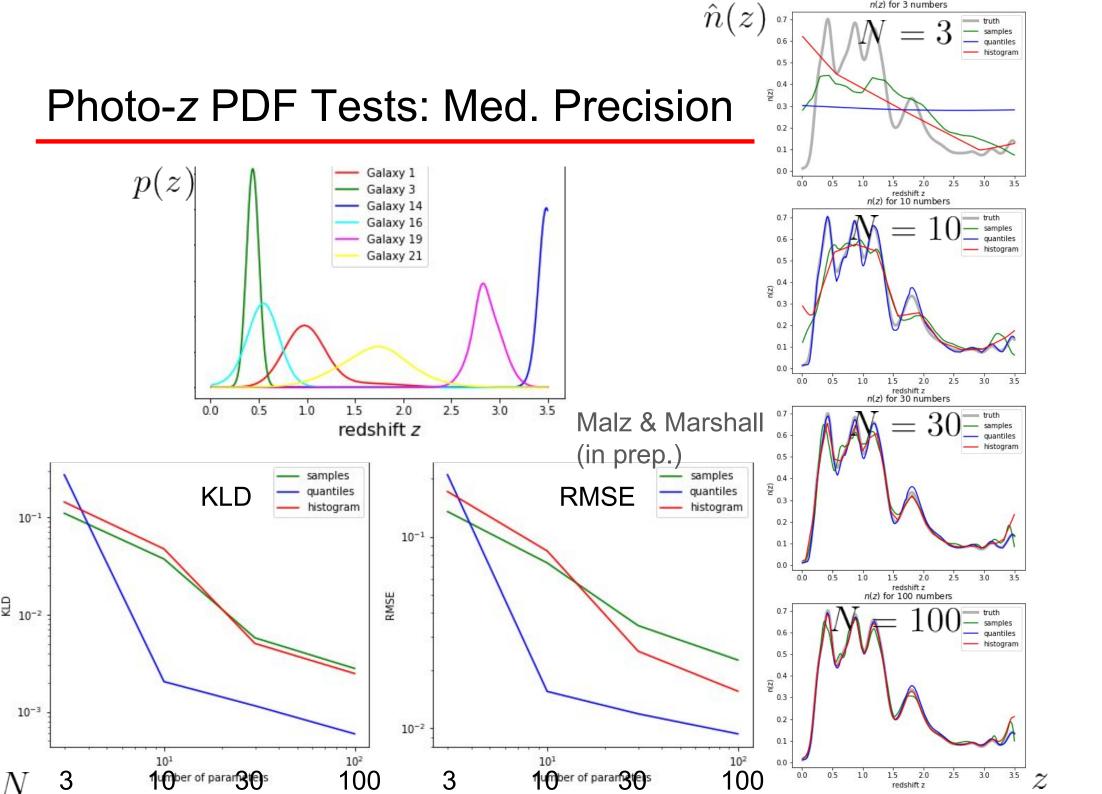
- Obtain realistically complex $\{p_r(z)\}_R$ with $\gg 200$ parameters
- Approximate by samples, binned, quantiles for different N
- Compare science case metrics over parametrizations and N

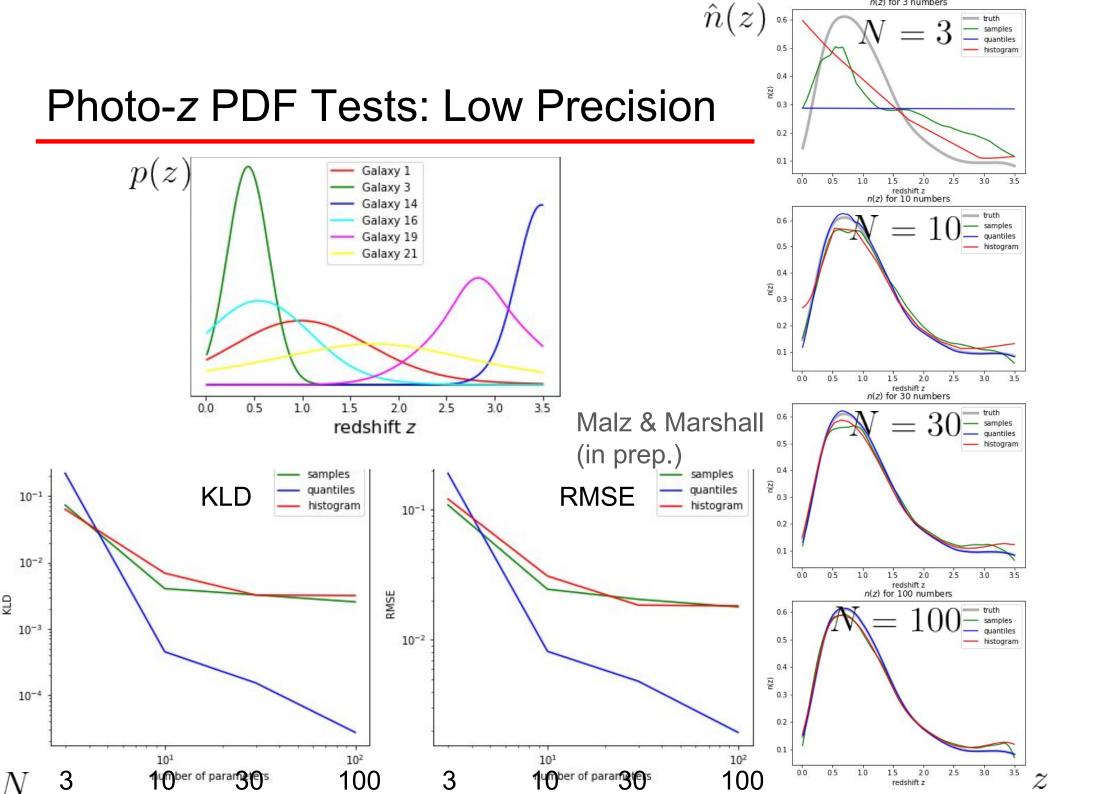
• Calculate
$$\hat{n}_p^{\bigstar}(z) = \sum_r^R p_r(z)$$
 and all $\hat{n}_{\hat{p}}(z) = \sum_r^R \hat{p}_r(z)$

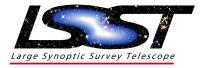
• Compute metrics (RMSE & KLD) for all cases

*** Caveat**: The "stacked" $\hat{n}(z)$ is <u>not</u> a valid estimator for the redshift distribution n(z) !









(Malz & Marshall, in prep.)

Features (existing, under development, planned)

- Parametrizations: samples, bins, quantiles, grid, GMM, *MM
 - Conversions to quantify effect on science cases
- $\hat{p}(z)$ Metrics: RMSE, KLD, KS, CvM, AD, QQ, moments
 - Tutorials for each metric to provide intuition for interpretation
- Ensembles: metrics applied to science cases

 \circ stacked $\hat{n}(z)$, other science uses of $\hat{p}(z)$

https://github.com/aimalz/qp